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CHEMICAL ENGINEERING
19/ENG01/013
SERIAL NUMBER: 48
MAT 104 Assignment

Question 1

Evaluate dy/dx at $x = 2.5$, correct to 3 significant figures given $y = (2x^2+3)/\ln 2x$

Solution

$u = 2x^2 + 3$ $\quad$ $v = \ln 2x$

$du/dx = 4x$ $\quad$ $dv/dx = 1/x$

$$dy/dx = \frac{V\,du/dx - U\,dv/dx}{V^2}$$

$$dy/dx = \frac{(\ln 2x)4x - 1/x(2x^2+3)}{(\ln 2x)^2}$$

$$dy/dx = \frac{4x\ln 2x - 2x + 3/x}{(\ln 2x)^2}$$

$$dy/dx = \frac{4x\ln 2x}{(\ln 2x)(\ln 2x)} - \frac{\frac{2x^2+3}{x}}{(\ln 2x)^2}$$

$$\frac{dy}{dx} = \frac{4x}{\ln 2x} - \frac{2x^2+3}{x(\ln 2x)^2}$$

$$\frac{dy}{dx} \text{ at } x = 2.5 \Rightarrow \frac{4(2.5)}{\ln 2(2.5)} - \frac{2(2.5)^2+3}{2.5(\ln 2(2.5))^2}$$

$$= \frac{10}{1.609} - \frac{15.5}{6.476}$$

$\therefore \frac{dy}{dx}$ at $x = 2.5 \Rightarrow 3.82$

Question 2

Find the gradient of the curve $y = \frac{2x}{(x^2-5)}$ at the point $(2, -4)$

Solution

Gradient of $y = dy/dx$

$u = 2x$ $\quad$ $v = x^2 - 5$

$du/dx = 2$ $\quad$ $dv/dx = 2x$

$$dy/dx = \frac{V\,du/dx - u\,dv/dx}{V^2}$$

$$\frac{dy}{dx} = \frac{2(x^2-5) - 2x(2x)}{(x^2-5)^2}$$

$$\frac{dy}{dx} = \frac{2x^2 - 10 - 4x^2}{(x^2-5)^2}$$

$$\frac{dy}{dx} = \frac{-2x^2 - 10}{(x^2-5)^2}$$

Gradient at point $(2, -4) = \frac{-2(2)^2 - 10}{(2^2-5)^2}$

$$= \frac{-8-10}{(-1)^2} = \frac{-18}{1}$$

$\therefore$ Gradient of curve y at point $(2, -4) = -18$

Question 3

If $z = 2x^3 \ln y$ find dz/dy.

Solution

$$\frac{dz}{dy} = \frac{d}{dy}(2x^3 \ln y)$$

$$\frac{dz}{dy} = \left(6x^2 \frac{dx}{dy} \cdot \ln y\right) + \left(\frac{1}{y} \cdot 2x^3\right)$$

$$\boxed{\frac{dz}{dy} = \frac{6x^2 \ln y \, dx}{dy} + \frac{2x^3}{y}}$$

Question 4

Integrate $x(2x^2+1)^{1/2}$ with respect to x from 0 to 2

Solution

$$\int_0^2 x(2x^2+1)^{1/2} \, dx$$

$$u = (2x^2+1)^{1/2}$$

$$u^2 = 2x^2+1$$

$$u^2+1 = 2x^2$$

$$\frac{u^2+1}{2} = x^2$$

$$x = \left(\frac{u^2+1}{2}\right)^{1/2}$$

$$\frac{dx}{du} = \frac{1}{2}\left(\frac{u^2+1}{2}\right)^{-1/2} \cdot 2u$$

$$\frac{dx}{du} = u\left(\frac{u^2+1}{2}\right)^{-1/2}$$

$$\frac{dx}{du} = u\left(\frac{2}{u^2+1}\right)^{1/2}$$

$$dx = u\left(\frac{u^2+1}{2}\right)^{-1/2} du$$

$$\int_0^2 \left(\frac{u^2+1}{2}\right)^{1/2} \cdot u \cdot u \cdot \left(\frac{u^2+1}{2}\right)^{-1/2} du$$

$$\int_0^2 u^2 \, du$$

$$\int_0^2 u^2 \, du$$

$$\frac{u^3}{3}\Big|_0^2$$

$$\frac{(2x+1)^{1/2 \times 3}}{3}\Big|_0^2$$

$$\frac{(2x+1)^{3/2}}{3}\Big|_0^2$$

$$\frac{(2(2)+1)^{3/2}}{3} - \frac{(2(0)+1)^{3/2}}{3}$$

$$3.73 - 0.33 = 3.4$$

$$\therefore \int_0^2 x(2x+1)^{1/2} \, dx = 3.4$$